# The Interchange of Source and Detector in Low-Power Microwave Network Measurements

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*Abstract*—The technique for interchanging generator and detector in the impedance measurement of microwave one-ports is a useful, known procedure often applied when low powers are indicated. The necessary and sufficient conditions for the validity of such measurements are examined critically and direct extensions of this technique to measurements of reciprocal two-ports are given.

A completely separate analysis is necessary when such an interchange is made in the case of an interference bridge which is to be used for the determination of the scattering parameters of arbitrary (active or passive, and reciprocal or nonreciprocal) two-ports. This analysis, presented in detail, results in a new low-power-level version of a method of measuring arbitrary two-ports as outlined in an earlier paper [5]. The measurement technique and the subsequent data analysis of the two versions are found to be identical, except that the two scattering parameters $S_{12}$ and $S_{21}$ appear in interchanged positions.

## I. Introduction

IN THE (one-port) measurement of the impedance characteristics of devices such as diodes for which powers may have to be held below certain maximum levels, it has become common practice to interchange generator and detector in order to increase the volume of power to the detector substantially above the level it would reach in a normal impedance measuring setup [1]. This approach has been suggested with regard to bridge techniques for the measurement of two-port parameters under similar limitations of maximum allowable power levels. In particular, it was felt that such an approach would be useful in certain measurements which seek to avoid regions of saturation or of nonlinearity. A case in point has been a measurement which tries to determine the parameters of a two-port cavity maser with an exceptionally small filling factor operating in its linear region, i.e., at very-low-power levels. The present work, though applicable in many situations where greater sensitivity is desired, was carried out in connection with this experiment and resulted in an increase in sensitivity of some 20 to 30 dB.

The standard one-port technique first is briefly reviewed and then more critically examined. Its extension to two-port impedance measurements also is indicated. The technique to be used in a reciprocal bridge for the measurement of reciprocal two-ports is indicated; finally, the method associated with source-detector interchange in the measurement of arbitrary (potentially nonreciprocal and active) linear two-ports by means of a practical interference bridge—one which also contains isolators—is derived. For this last method, a step-by-step summary of the measurement is offered in Appendix III.

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## II. Interchange in Impedance Measurements

### A. One-Port Measurements

The common technique for measuring the impedance of an unknown load is depicted in Fig. 1. The idea of a matched detector here implies that while energy may be reflected by the probe, or even by some structure located between the probe and the detector, no microwave energy may be reflected by the detector itself. Both the detector and the generator may be taken to include any necessary tuners or isolators. The structure contained within dashed lines in Fig. 1, including the unknown load, can be regarded as a reciprocal two-port between terminal planes $T_1$ and $T_2$. Its parameters, of course, depend upon the load and the location of the probe. In view of the reciprocity of the two-port ($S_{12}=S_{21}$), and the match of the generator and the detector, the reading on the indicator remains completely unaltered when these two components are interchanged as in Fig. 2. Since this is true for any similar two-port, i.e., for any load or any probe position, measurements may be carried out exactly as though the interchange had not been made.

In Fig. 1 the generator power has been set at a level $p$ (say, in dBm), so that power at the probe and the load is also very nearly at $p$; however, in view of a dB decoupling of the probe, power at the detector is $(p-\alpha)$. In contrast, in Fig. 2, generator power is $(p+\alpha)$ so that the level at the probe and the load is again $p$ as it was before, but power at the detector no longer is $(p-\alpha)$ but $p$.

The virtue of this method then lies in the fact that, while the power level at the load was maintained within imposed bounds, the level at the detector has been increased by $\alpha$ dB, where $\alpha$ presumably represents some minimum probe decoupling. Advantage can be taken of any additional available generator power by further decoupling the probe as far as this power will permit. Two distinct advantages are now possible: additional sensitivity of $\alpha$ dB, and the greater accuracy which results from the reduction of the usual errors associated with probe reflection.

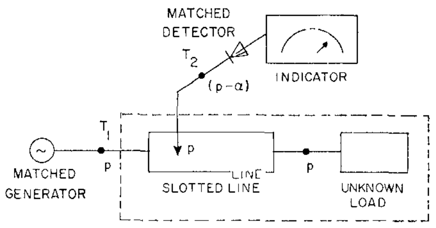


Fig. 1. Common impedance measurement.

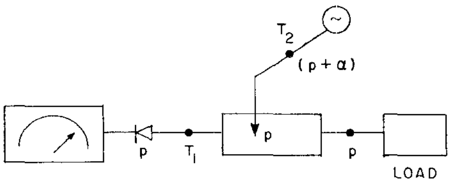


Fig. 2. Source-detector reversal in load impedance measurement.

As is to be expected, a price must be paid for what has been achieved. When the probe is moved in the course of the measurement, a flexible waveguide must be used in Fig. 2 instead of the audio cable of Fig. 1. The flexible waveguide may introduce unwanted leakage or additional small probe-position dependent reflections. When this becomes intolerable two microwave-wise sound but mechanically troublesome alternatives have been used: either the generator (tube) can be connected rigidly to the probe carriage and moved with it, or the generator and probe can be held fixed while the slotted line, detector, and load are moved as a unit with respect to the probe.

*B. Extension to Two-Port Impedance Measurements*

A useful feature of this scheme is the fact that *any* measurement technique involving equipment which can somehow be viewed as a reciprocal microwave two-port connected to a matched source at one port, and to a matched detector at the other, is subject to source-detector interchange without in the least affecting the method of measurement or the associated data analysis. Further thought is not required, nor need any new formulas be derived. For example, the Weissfloch [2] method of measuring reciprocal two-ports,[1] or the Deschamps [3] method of finding the scattering coefficients of reciprocal two-ports can be used, and perhaps to advantage, with source and detector interchanged.

*C. Necessary and Sufficient Conditions for Source-Detector Interchange*

Although the restriction of matched generator and detector undoubtedly is sufficient, the necessary conditions should also be explored. As will be seen, these somewhat extend the usefulness of the technique. Consider a nonreciprocal two-port connected to a generator with internal impedance $Z_g$ and to a detector of impedance $Z_d$, as in Fig. 3. Detector response is proportional to $(I_d/V_g)^2$ so that source and detector may be interchanged provided

$$\left[\frac{I_d/V_g}{I_d'/V_g}\right]^2 = \text{constant} \tag{1}$$

as the two-port parameters are varied in the course of a measurement. Here $I_d'$ is the detector current after the interchange has been made. Clearly, from (1)

$$\frac{I_d/V_g}{I_d'/V_g} = \text{constant} \tag{2}$$

is also a necessary and sufficient condition. It is convenient to recognize that $Z_g$ and $Z_d$ are associated with the diagonal elements of the impedance matrix as in Fig. 4, which indicates only the relevant voltages and currents. From Fig. 4, one has

$$\begin{aligned} V_g &= (Z_{11} + Z_g)I_g + Z_{12}I_g \\ V &= 0 = Z_{21}I_g + (Z_{22} + Z_d)I_d \end{aligned} \tag{3}$$

and when $I_g$ is eliminated from (3), the result is

$$I_d/V_g = \frac{Z_{21}}{(Z_{12}Z_{21} - Z_{11}Z_{22} - Z_dZ_g) - (Z_{11}Z_d + Z_{22}Z_g)} \tag{4}$$

From Fig. 2, it is apparent that the interchange can be represented simply by interchanging subscripts 1 and 2, so that from (2) and (4) the desired condition is

$$\frac{Z_{21}}{Z_{12}} \cdot \frac{(Z_{12}Z_{21} - Z_{11}Z_{22} - Z_dZ_g) - (Z_{22}Z_d + Z_{11}Z_g)}{(Z_{12}Z_{21} - Z_{11}Z_{22} - Z_dZ_g) - (Z_{11}Z_d + Z_{22}Z_g)} = \text{constant} \tag{5}$$

Since the two-port parameters have been assumed to be arbitrary, and since their variation in the course of a measurement has not been restricted, (5) indicates that the necessary and sufficient condition for source-detector interchange is

$$Z_d = Z_g, \text{ provided } Z_{21}/Z_{12} = \text{constant} \tag{6}$$

This proviso holds in most practical measurement situations in view of the theorem presented in Appendix I:

> The product of the nonreciprocities of tandem two-ports equals the nonreciprocity of the overall two-port.

The nonreciprocity of any two-port $\mathfrak{z}$ here is taken as $\sqrt{\mathfrak{z}_{12}/\mathfrak{z}_{21}}$. The nonreciprocity of a reciprocal two-port is, of course, unity. It is clear then that if the two-port $Z$ is comprised of an arbitrary number of cascaded two-ports, $Z_{21}/Z_{12}$ is constant as long as all two-ports in the chain with parameters that change in the course of the measurements are reciprocal. An example of such a situation would be an isolator at $T_1$ in tandem with the slotted line in Figs. 1 and 2.

[1] These, by the way, may be active if the first resistance in the Weissfloch representation is appropriately taken as either positive or negative. The distinction between reflection coefficients greater and less than unity must, however, somehow be made in the course of the measurement.

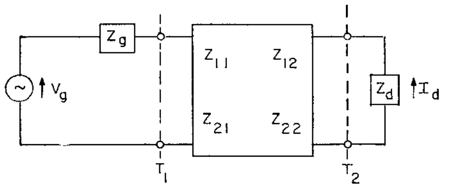


Fig. 3. Two-port with source and detector.

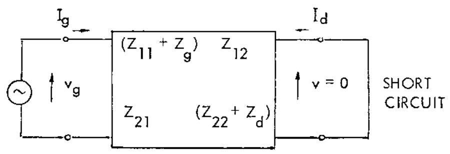


Fig. 4. Two-port incorporating generator and detector impedances.

It can be shown quite independently that when (6) is recast in scattering terms, it becomes

$$\Gamma_d = \Gamma_g, \qquad \text{provided } S_{21}/S_{12} = \text{constant} \tag{7}$$

where $\Gamma_d$ and $\Gamma_g$ are the reflection coefficients of the detector and the generator, respectively. This, of course, also follows simply from the recognition that $\Gamma_d$ and $\Gamma_g$ are the input reflection coefficients corresponding to the detector and generator input impedances $Z_d$ and $Z_g$ and from the fact [4] that $S_{21}/S_{12}=Z_{21}/Z_{12}$ for any two-port.

Equations (5) to (7) are quite general and as such apply also to the interchange pictured in Figs. 1 and 2 involving a slotted line. When, in that case, a properly tuned and well-decoupled probe is employed, the following approximations usually can be made:

$Z_{11}$ is constant as the probe is moved;
$Z_{22}$ is constant as the probe is moved;
$Z_{12}Z_{21}$ is sufficiently small to be neglected.

Under these conditions, it is seen that (5) holds without any restrictions having been placed on $Z_g$ and $Z_d$, again provided, that the ratio $Z_{21}/Z_{12}$ is a constant as explained previously.

## III. Interchange in Bridge Measurements

### A. Reciprocal Case

Microwave bridge measurements are used in situations involving nonreciprocal as well as reciprocal two-ports as unknowns. When the latter is the case, and when the bridge itself, as distinct from the detector and the generator arms, does not contain nonreciprocal components, the results given for the general case in (6) and (7) are, of course, seen to hold as soon as a two-port connecting source and detector can be identified. Figure 5, which illustrates a simplified outline of an interference bridge, identifies this two-port as being located between planes $T_1$ and $T_2$.

### B. Nonreciprocal Case

When a microwave bridge has internal nonreciprocal components, such as isolators, or when it is used for the

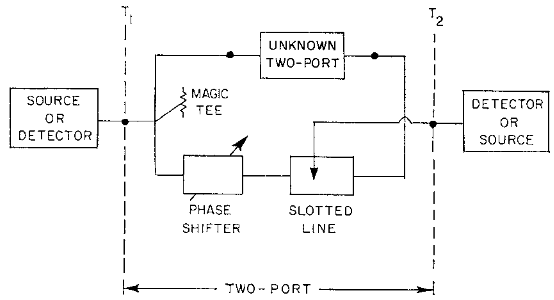


Fig. 5. Source-detector reversal with reciprocal bridge.

measurement of nonreciprocal structures, the preceding reciprocity argument can no longer be used when source and detector are interchanged. However, one may seek the answer to the question of what actually does happen when the interchange is made in such a case. This is done for an interference bridge. The measurement which evolves will be seen to have the same advantages with respect to power levels as the corresponding impedance measurement discussed earlier.

The two forms of the bridge which are sketched in Figs. 6 and 7 will be referred to as the "forward" bridge and the "reversed" bridge, respectively. Note that the generator (including the associated isolator) and the detector have been interchanged, and that the two isolators adjacent to the slotted lines have been reversed. In each setup, only one probe at a time is in use. It should also be noted that the regions in which standing waves exist, and those in which traveling waves exist, are quite different in the two bridges. Since the forward bridge has already been amply treated [5], it will not be discussed here.

*1) Parameters of the Data Loci:* Consider the reversed bridge with the source connected to the left probe for the moment. Two waves $a_1'$ and $a_2'$ are set up in the left slotted line by the probe and travel in the directions indicated in the figure. As a result, three distinct waves reach and add at the detector. The voltage wave traveling from the probe directly to the left is

$$V_1 = a_1' C_1 e^{j\kappa z_1} \tag{8}$$

where $C_1$ is the overall transmission coefficient through the left bridge arm to the detector arm and includes the effect of the variable phase shifter $\phi$ and the variable attenuator $A$. The wave from the probe to the right which is transmitted through the two-port is

$$V_2 = a_2' C_2 e^{-j\kappa z_1} S_{21} \tag{9}$$

where $C_2$ is analogous to $C_1$, but a constant. The wave from the probe to the right which is reflected by the two-port is

$$V_3 = a_2' C_1 e^{-j\kappa z_1} S_{11} \tag{10}$$

The voltage at the input of the detector, $V_d$, then is the sum of $V_1$, $V_2$ and $V_3$

$$V_d = a_2' e^{-j\kappa z_1}(S_{11}C_1 + S_{21}C_2) + a_1' e^{j\kappa z_1} C_1 \tag{11}$$

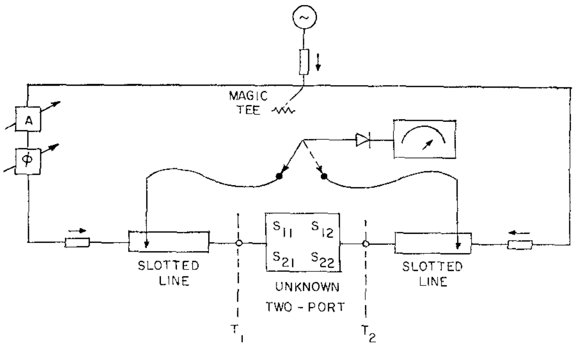


Fig. 6. Forward bridge.

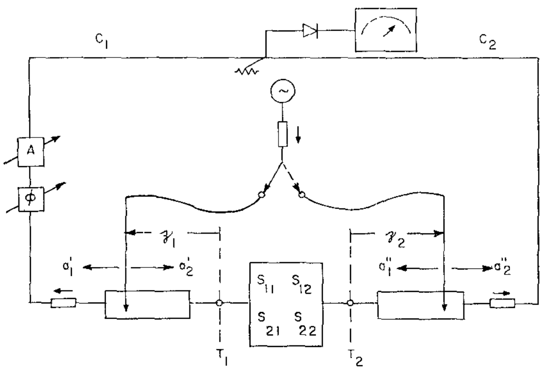


Fig. 7. Reversed bridge.

which can also be written

$$V_d = a_1'C_1\left[e^{-j\kappa z_1}\frac{a_2'}{a_1'}\left(S_{11}+\frac{C_2}{C_1}S_{21}\right)+e^{j\kappa z_1}\right] \quad (12)$$

Since a voltage which need only be proportional to the magnitude of $V_d$ is of interest, it is sufficient to consider

$$V_d' = e^{-j\kappa z_1}\frac{a_2'}{a_1'}\left(S_{11}+\frac{C_2}{C_1}S_{21}\right)+e^{j\kappa z_1} \quad (13)$$

In view of the definition of $z_1$ (increasing from right to left in Fig. 7), the exp ($k\kappa z_1$) and exp ($-j\kappa z_1$) terms, respectively, are incident (from the left) on plane $T_1$ and "reflected" at plane $T_1$. The coefficient of exp ($-j\kappa z_1$) consequently is identified as the reflection coefficient $\Gamma_1$ at $T_1$ (defined, looking to the right). As phase shifter $\phi$ is varied, the phase of $C_1$ varies with it, and $\Gamma_1$ is seen to be a circular locus. If the probe is perfectly symmetrical, i.e., straight, as one attempts to make it, then $a_1'=a_2'$ and

$$\Gamma_1 = S_{11}+\frac{C_2}{C_1}S_{21} \quad (14)$$

The conditions under which $a_1'=a_2'$ are discussed in Appendix II. As in the case of the forward bridge, the $\Gamma_1$ locus has its center at $S_{11}$, but it now has the radius $|S_{21}|$, provided the bridge has been balanced (to make $|C_1|=|C_2|$) by adjustment of variable attenuator $A$.

Analogously, for probe 2, with $z_2$ defined as increasing to the right

$$V_d'' = e^{-j\kappa z_2}\frac{a_1''}{a_2''}\left(S_{22}+\frac{C_1}{C_2}S_{12}\right)+e^{j\kappa z_2} \quad (15)$$

and

$$\Gamma_2 = S_{22}+\frac{C_1}{C_2}S_{12} \quad (16)$$

where $\Gamma_2$ is the reflection coefficient at plane $T_2$ defined looking to the left. The locus of $\Gamma_2$ again is a circle, but with center at $S_{22}$ and, when the bridge has been balanced, with radius $|S_{12}|$.

In terms of traveling waves, the balance condition of a reversed bridge is the following: if $a_1$ equals $a_2$, and $a_1$ is defined at $T_1$ and traveling to the left, while $a_2$ is defined at $T_2$ and traveling to the right, then, if detector voltage is zero when these two waves exist at the same time, the bridge is balanced, i.e., $|C_1|=|C_2|$. Some reflection shows that the various ways of balancing a forward bridge [5] apply completely in this case as well.

2) *Reference Plane and Phase Measurements:* When the scale with which $z_1$ is measured is the $D$ scale so that

$$z_1 = D - D_R \quad (17)$$

the reference value $D_R$, which is defined with respect to $T_1$, is found in the same fashion as if the bridge were a forward bridge. This follows readily from the fact that such measurements are not any longer actual bridge measurements, but are impedance measurements of the sort discussed earlier. Similarly, when the scale with which $z_2$ is measured is the $S$ scale so that

$$z_2 = S - S_R \quad (18)$$

the reference value $S_R$, which is defined with respect to $T_2$, again is found as in a forward bridge.

Equations (13) and (15) are both of the form

$$V_d = e^{-j\kappa z}\Gamma + e^{j\kappa z}, \qquad \Gamma \equiv |\Gamma| e^{j\theta} \quad (19)$$

The determination of such a value of $\Gamma$ with a reversed bridge will now be considered. The probe can be moved to two distinct locations such that $|V_d|$ is a maximum ($V_M$) or a minimum ($V_m$). Let the probe setting associated with $V_m$ be $z_m$. Then

$$V_M = |\Gamma| + 1 \quad (20)$$

$$V_m = ||\Gamma| - 1| = ||\Gamma| e^{j(\theta-\kappa z_m)} + e^{j\kappa z_m}| \quad (21)$$

so that from (21)

$$\theta = 2\kappa z_m \pm \pi \quad (22)$$

But, since $z_1$ and $z_2$ are given by (17) and (18), we obtain

$$\theta_1 = 2\kappa(D - D_R) \pm \pi \quad (23)$$

$$\theta_2 = 2\kappa(S - S_R) \pm \pi \quad (24)$$

where $D$ and $S$ must now be interpreted as the probe positions, when $V_d'$ and $V_d''$ are minimum. From (20) and (21)

$$|\Gamma| = \frac{r-1}{r+1}, \qquad r \equiv \pm V_M/V_m \tag{25}$$

Since $|\Gamma|$ may either be greater or less than unity, the standing-wave ratio, $r$, is taken to be positive for $|\Gamma| < 1$, and negative for $|\Gamma| > 1$. Means of deciding whether $|\Gamma|$ is greater or less than unity, which also apply here, are given elsewhere [5].

When the unknown two-port of Fig. 7 is replaced by a waveguide of length $l_0$ so that no reflections occur at $T_1$ or at $T_2$, and when the phase shifter is set at some arbitrary but fixed setting $\phi_0$, then each probe in turn can be moved to such a position that there will be minimum detector output (zero when $a_1/a_2=1$ and $|C_1/C_2|=1$). Let these probe settings be defined as $D_0$ and $S_0$. In view of the fact that the scattering matrix of line $l_0$ is

$$\begin{pmatrix} 0 & e^{-j\kappa l_0} \\ e^{-j\kappa l_0} & 0 \end{pmatrix} \tag{26}$$

and with the assumption that $a_1=a_2$, detector voltages $V_d$ in (13) and (15) will be at a minimum when

$$e^{-j\kappa(D_0-D_R)}e^{j(\phi_2-\phi_{10}-\kappa l_0)} = -e^{j\kappa(D_0-D_R)} \tag{27}$$

and when

$$e^{-j\kappa(S_0-S_R)}e^{j(\phi_{10}-\phi_2-\kappa l_0)} = -e^{j\kappa(S_0-S_R)} \tag{28}$$

where $C_2$ has the phase $\phi_2$ and $C_1$ the phase $\phi_1$. When, as now is the case, the phase shifter is set to $\phi_0$, the angle $\phi_1$ takes on the value $\phi_{10}$. From (27) and (28), one readily obtains

$$\phi_2 - \phi_{10} = 2\kappa(D_0 - D_R) + \kappa l_0 \pm \pi \tag{29}$$

$$\phi_2 - \phi_{10} = -[2\kappa(S_0 - S_R) + \kappa l_0 \pm \pi] \tag{30}$$

Equating (22) and (23), one finds that

$$D_0 + S_0 + l_0 - D_R - S_R = \frac{n\lambda_g}{2} \tag{31}$$

Equation (31) can serve as a consistency check on the measurements made in finding $D_0$, $S_0$, $l_0$, $D_R$, and $S_R$.

It is convenient to plot the measured data in the $\bar{\Gamma}_1$ and $\bar{\Gamma}_2$ planes where

$$\bar{\Gamma}_1 \equiv |\Gamma_1| e^{j2\kappa D} = |\Gamma_1| e^{j\bar{\theta}_1} = \Gamma_1 e^{j(2\kappa D_R \pm \pi)} \tag{32}$$

$$\bar{\Gamma}_2 \equiv |\Gamma_2| e^{j2\kappa S} = |\Gamma_2| e^{j\bar{\theta}_2} = \Gamma_2 e^{j(2\kappa S_R \pm \pi)} \tag{33}$$

It follows from these equations and from (14) and (16) that

$$|S_{11}| e^{j(\phi_{11}+2\kappa D_R \pm \pi)} + \left|\frac{C_2}{C_1} S_{21}\right| e^{j(\phi_{21}+2\kappa D_R \pm \pi + \phi_2 - \phi_1)}$$

$$\equiv |S_{11}| e^{j\bar{\phi}_{11}} + \left|\frac{C_2}{C_1} S_{21}\right| e^{j\psi_1} \tag{34}$$

and that

$$\bar{\Gamma}_2 = |S_{22}| e^{j(\phi_{22}+2\kappa S_R \pm \pi)} + \left|\frac{C_1}{C_2} S_{12}\right| e^{j(\phi_{12}+2\kappa S_R \pm \pi - \phi_2 + \phi_1)}$$

$$\equiv |S_{22}| e^{j\bar{\phi}_{22}} + \left|\frac{C_1}{C_2} S_{12}\right| e^{j\psi_2} \tag{35}$$

and, therefore, it is easy to identify

$$\phi_{11} = \bar{\phi}_{11} - 2\kappa D_R \pm \pi \tag{36}$$

$$\phi_{22} = \bar{\phi}_{22} - 2\kappa S_R \pm \pi \tag{37}$$

When the two-port to be measured is located in the bridge, and the phase shifter is set to the same value $\phi_0$ which was used earlier, $\phi_1$ again takes on the value $\phi_{10}$, the angles $\psi_1$ and $\psi_2$ become the specific angles $\psi_{10}$ and $\psi_{20}$, and $\bar{\Gamma}_1$ and $\bar{\Gamma}_2$ are the measured points $\bar{\Gamma}_{10}$ and $\bar{\Gamma}_{20}$. Under these conditions one identifies

$$\phi_{21} = \psi_{10} - 2\kappa D_R \pm \pi - \phi_2 + \phi_{10} \tag{38}$$

$$\phi_{12} = \psi_{20} - 2\kappa S_R \pm \pi - \phi_2 - \phi_{10}. \tag{39}$$

On using (29) and (30) in (38) and (39), respectively, one has

$$\phi_{21} = \psi_{10} - 2\kappa D_0 - \kappa l_0 \tag{40}$$

$$\phi_{12} = \psi_{20} - 2\kappa S_0 - \kappa l_0 \tag{41}$$

Figure 8 shows the $\bar{\Gamma}_1$ and $\bar{\Gamma}_2$ loci explicitly in the sense of (34) and (35). The points $\bar{\Gamma}_{10}$ and $\bar{\Gamma}_{20}$ and the associated angles $\psi_{10}$ and $\psi_{20}$ also are shown. The radii, of course, yield $|S_{21}|$ and $|S_{12}|$ directly when the bridge has been balanced ($|C_1|=|C_2|$).

The measurement procedure to be used with the reversed bridge is summarized in Appendix III.

*3. Comparison of Measurements with Forward and Reversed Bridges:* When we compare the analysis of measurements with a forward bridge with that made with a reversed bridge, we find that the following simple generalization can be made: if a measurement and data analysis has been carried out with one bridge to determine the scattering matrix parameters of a two-port, then, if *precisely* the same measurement and analytical steps are followed with the other bridge, the same matrix elements are obtained, but in transposed order, i.e., $\tilde{S}$ instead of $S$. In other words, $|S_{11}|$, $\phi_{11}$, $|S_{22}|$, and $\phi_{22}$ result from the same steps in both cases, but the steps which produce $|S_{12}|$ and $\phi_{12}$ in one case produce $|S_{21}|$ and $\phi_{21}$ in the other.

*4. Reversed Bridge and Various Two-Port Representations:* Since, when matrices $Z$ or $Y$ correspond to $S$ then $\tilde{Z}$ or $\tilde{Y}$ correspond to $\tilde{S}$, we can make the following statement: any measurements performed with a forward bridge of the type considered to result in $Z$ or $Y$ will consequently result in $\tilde{Z}$ or $\tilde{Y}$ when performed with a reversed bridge. If other nonreciprocal representations, such as the scattering transfer matrix, the voltage-current transfer matrix ($ABCD$), or the nonreciprocal modified Wheeler network, can be obtained by the use of such bridges, they are not subject to similar matrix

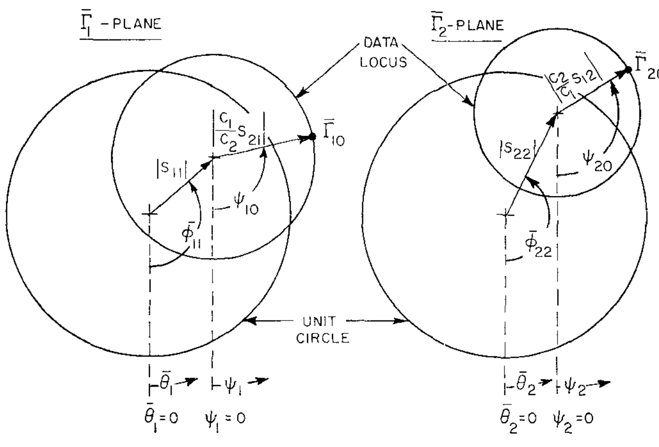


Fig. 8. Reflection coefficient loci obtained with reversed bridge.

transposition. Instead, it is recognized that the nonreciprocal character of such a representation is described by its nonreciprocity, $K$, and that the notion of the transposition of the $S$ matrix of a two-port is completely described by the inversion of the nonreciprocity [see (44)]. This notion, therefore, can be used whenever matrix transposition is not any longer in place. For example, in the case of the $ABCD$ matrix [see (42)], if $A$, $B$, $C$, and $D$ were the parameters obtained by means of a forward bridge, the same measurement with a reversed bridge would yield $A/K^2$, $B/K^2$, $C/K^2$, and $D/K^2$. The converse, of course, is also true.

## IV. Conclusions

The interchange of source and detector in microwave impedance and bridge measurements results in increased sensitivity and, depending on available power, makes the reduction of probe error possible. This technique can be applied to one-port and any two-port impedance measurement. The restrictions of matched generator and matched load have been shown to be too stringent, and certain nonreciprocal components may be employed in the measuring circuits.

Bridges which do not contain nonreciprocal components may be subjected to source-detector interchange in the measurement of reciprocal two-ports. The interchange also has been shown to be possible in the case of a specific bridge containing isolators, which is used for the measurement of arbitrary two-ports. The measurement which evolves is very closely related to that with source and detector in their normal locations.

Measurements with source and detector interchanged depend on the availability of additional power. With such power available, and assuming the power at the load or the two-port being measured to be fixed, power at the detector can be increased by the number of dB representing the probe (or directional coupler) decoupling. These facts make this arrangement especially suitable for measurements of components which, for any reason, must be limited in power in the course of the measurement. Under such circumstances, the additional power required is usually available.

## Appendix I

### Product of Nonreciprocities

It has been shown [6] in voltage-current transfer matrix terms that a nonreciprocal two-port can be decomposed into a reciprocal two-port and a special nonreciprocal two-port (ratio repeater) as follows:

$$M \equiv \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} A_0 & B_0 \\ C_0 & D_0 \end{pmatrix} \begin{pmatrix} K & 0 \\ 0 & K \end{pmatrix} \equiv M_0 \mathcal{S} \qquad (42)$$

where $ABCD$ is the nonreciprocal two-port, and $A_0B_0C_0D_0$ the reciprocal two-port, i.e., where $\sqrt{A_0D_0 - B_0C_0} = 1$. The ratio repeater is represented by a scalar matrix $\mathcal{S}$ with the associated scalar $K \equiv \sqrt{AD - BC}$. Clearly, when several two-ports (indicated by different superscripts) are in tandem, then

$$M^1M^2 \cdots M^n = M_0{}^1\mathcal{S}^1 \cdot M_0{}^2\mathcal{S}^2 \cdots M_0{}^n\mathcal{S}^n$$
$$= M_0{}^1M_0{}^2 \cdots M_0{}^n\mathcal{S}^1\mathcal{S}^2 \cdots \mathcal{S}^n \qquad (43)$$

since the matrices $\mathcal{S}$ commute with all other matrices. The product $\mathcal{S}^1\mathcal{S}^2 \cdots \mathcal{S}^n$ is again a scalar matrix with the associated scalar $K^1K^2 \cdots K^n$. Here $K^1$, $K^2$, $\cdots$, $K^n$ are the nonreciprocities of the two-ports $M^1$, $M^2$, $\cdots$, $M^n$, respectively, and the product $K^1K^2 \cdots K^n$ is the nonreciprocity of the overall two-port $M^1M^2 \cdots M^n$ so that one has the theorem:

> The product of the nonreciprocities of tandem two-ports, equals the nonreciprocity of the "overall two-port."

It has been shown [4] that the nonreciprocity $K$ (which is simply a complex number) is related to the parameters of various matrix two-port representations as follows:

$$K = \sqrt{AD - BC} = \sqrt{S_{12}/S_{21}} = \sqrt{Z_{12}/Z_{21}}$$
$$= \sqrt{Y_{12}/Y_{21}} = \sqrt{\tau_{11}\tau_{22} - \tau_{12}\tau_{21}} \qquad (44)$$

As usual, $S$ stands for scattering, $Z$ for impedance, $Y$ for admittance, and $\tau$ for scattering transfer. The theorem covering the product of nonreciprocities consequently is applicable to all these representations.

## Appendix II

### Probe Excited Traveling Waves

In considering a reversed bridge, it is necessary to have a clear picture of the circumstances under which $a_1'$ and $a_2'$, the two traveling waves set up by a voltage probe in a waveguide, can be considered to be equal. When the probe is either perfectly straight, or is bent in such a manner that it lies completely in the transverse plane, there is no problem—the two waves are equal. When the probe is bent partially or completely in the direction of propagation, the equality cannot be assumed.

Consider the following Gedanken experiment with reference to Fig. 9. Let both source and detector be matched and let the bridge be reciprocal and perfectly

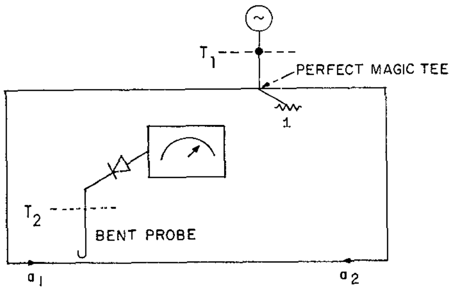


Fig. 9. Symmetric bridge with bent probe.

symmetrical about the magic tee except for the shape and location of the probe. The waves incident from the magic tee, $a_1$ and $a_2$, are consequently equal. Now move the probe to a voltage minimum. If the probe were straight, this would be a voltage zero to within the noise of the system. If the probe is bent, there still may exist an effective minimum location at which induced voltage is so small that it lies below the noise, or so slightly above the noise that it can be neglected. Under these circumstances, the detected voltage is, for all practical purposes, zero. This situation is relatively common in the usual slotted line measurements. Now, let source and detector be interchanged without moving the probe. The two-port between planes $T_1$ and $T_2$ is reciprocal so that the detected voltage again may be practically zero. This is possible only if the two waves incident on the magic tee are for practical purposes equal, which can be the case only if $a_1'$ and $a_2'$, the waves set up by the probe, have been very nearly equal. It is seen then that the two traveling waves set up by a voltage probe can be considered to be equal as long as the same probe is capable of measuring a voltage zero to within the desired accuracy or within the noise of the system.

## Appendix III

### Summary of Measurement Procedure for Reversed Bridge

1) Short-circuit planes $T_1$ and $T_2$, and measure $D_R$, $S_R$ and $\lambda_g$ (see Fig. 7).

2) Measure the length of a suitable waveguide, $l_0$, and insert it between planes $T_1$ and $T_2$. Balance the bridge [5].

3) Set the variable phase shifter to some arbitrary setting $\phi_0$, and measure $D_0$ and $S_0$.

4) The values found in the foregoing should satisfy the relation

$$D_0 + S_0 + l_0 - D_R - S_R = n\lambda_g/2, \quad n = 0, \pm 1, \pm 1, \cdots$$

5) Replace the waveguide $l_0$ by the two-port to be measured, and for each of a series of settings of $\phi$, measure $D$ and $r_1$ with the left slotted line and $S$ and $r_2$ with the right slotted line. Include the setting $\phi_0$ in the series. [Take $r_1$ and $r_2$, the VSWR (voltage standing-wave radio), as positive only.]

6) Compute

$$\bar{\Gamma}_1 = \frac{r_1 - 1}{r_1 + 1} e^{j2\kappa D}$$

and

$$\bar{\Gamma}_2 = \frac{r_2 - 1}{r_2 + 1} e^{j2\kappa S}$$

$$\kappa = \frac{2\pi}{\lambda_g}$$

corresponding to each $\phi$ setting. The values corresponding to $\phi_0$ are designated $\bar{\Gamma}_{10}$ and $\bar{\Gamma}_{20}$. Values of $|\bar{\Gamma}_1|$ and $|\bar{\Gamma}_2|$ found in this way may either be the proper values or their inverses. Be sure to distinguish between $|\Gamma|$ and its inverse.

7) Plot the points $\bar{\Gamma}_1$ and $\bar{\Gamma}_2$ on two sheets of polar coordinate paper and draw a circle through each.

8) Identify $|S_{11}|$, $\bar{\phi}_{11}$, $\psi_{10}$, and $|S_{21}|$ (Note: $|C_1| = |C_2|$) on the $\bar{\Gamma}_1$ locus, and $|S_{22}|$, $\bar{\phi}_{22}$, $\psi_{20}$, and $|S_{12}|$ on the $\bar{\Gamma}_2$ locus. She Fig. 8 for details.

9) Compute

$$\phi_{11} = \bar{\phi}_{11} - 2\kappa D_R \pm \pi$$

$$\phi_{22} = \bar{\phi}_{22} - 2\kappa S_R \pm \pi$$

$$\phi_{21} = \psi_{10} - 2\kappa D_0 - \kappa l_0$$

$$\phi_{12} = \psi_{20} - 2\kappa S_0 - \kappa l_0$$

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